

# **MAT**

## **SERIE A : CONFERENCIAS, SEMINARIOS Y TRABAJOS DE MATEMÁTICA**

**No. 2**

### **A BIBLIOGRAPHY ON MOVING-FREE BOUNDARY PROBLEMS FOR THE HEAT-DIFFUSION EQUATION. THE STEFAN AND RELATED PROBLEMS**

**Domingo Alberto TARZIA**

Departamento de Matemática - CONICET,  
Facultad de Ciencias Empresariales, Universidad Austral,  
Paraguay 1950, S2000FZF Rosario, ARGENTINA.

E-mail: [tarzia@uafce.edu.ar](mailto:tarzia@uafce.edu.ar)

**Rosario, Julio 2000**

## ABSTRACT

We present a bibliography on moving and free boundary problems for the heat-diffusion equation, particularly regarding the Stefan and related problems.

It contains 5869 titles referring to 588 scientific Journals, 122 books, 88 symposia (having at least 3 contributions on the subject), 30 collections, 59 thesis and 247 technical reports.

It tries to give a comprehensive account of the western existing mathematical-physical-engineering literature on this research field.

## RESUMEN

Se presenta una bibliografía sobre problemas de frontera móvil y libre para la ecuación del calor-difusión, en particular sobre el problema de Stefan y problemas relacionados.

Contiene 5869 títulos distribuidos en 588 revistas científicas, 122 libros, 88 simposios (teniendo al menos 3 contribuciones en el tema), 30 colecciones, 59 tesis y 247 informes técnicos o prepublicaciones.

Se da un informe amplio de la bibliografía matemática, física y de las ingenierías existente en occidente sobre este tema de investigación.

**Primary Mathematics Subject Classification Number (\*)**: 35R35, 80A22

**Secondary Mathematics Subject Classification Number (\*)**: 35B40, 35C05, 35C15, 35Kxx, 35R30, 46N20, 49J20, 65Mxx, 65Nxx, 76R50, 76S05, 76T05, 93C20.

(\*) Following the 1991 Mathematics Subject Classification compiled by Mathematical Reviews and Zentralblatt fur Mathematik.

**Primary key words**: Enthalpy formulation or method, Filtration, Free boundary problems, Freezing, Melting, Moving boundary problems, Mushy region, Phase-change problem, Solidification, Stefan problem.

**Secondary key words**: Continuous mechanics, Diffusion process, Functional analysis, Heat conduction, Mathematical methods, Numerical methods, Partial differential equations, Variational inequalities, Weak solutions.

**Palabras claves primarias**: Método o formulación en entalpía, Filtración, Problemas de frontera libre, Congelación, Derretimiento, Problemas de frontera móvil, Región pastosa, Problema de cambio de fase, Solidificación, Problema de Stefan.

**Palabras claves secundarias**: Mecánica del continuo, Procesos difusivos, Análisis funcional, Conducción del calor, Métodos matemáticos, Métodos numéricos, Ecuaciones diferenciales a derivadas parciales, Inecuaciones variacionales, Soluciones débiles.

## I. INTRODUCTION.

This personal bibliography on moving and free boundary problems (M-FBP) for the heat-diffusion equation (H-DE) contains about 5900 references to works appeared on approximately 884 different kinds of publications. It tries to give a comprehensive account of the western existing mathematical-physical-engineering literature on this research field.

Almost all the material on the subject, published after the historical and first paper of Lamé-Clapeyron (1831), has been collected. Sources include scientific journals, symposium or conference proceedings, technical reports and books.

References quoted in Tarzia (1981, 1984, 1988) (773, 936 and 2528 references in the years 1981, 1984 and 1988 respectively) are included in the present bibliography.

This issue is a document to be up-to-dated by addition of new references with the purpose of making a data-base and maybe a classification similar to the one done in the 1981 bibliography.

All the papers are directly related to some aspects of the M-FBP for the H-DE, particularly regarding the phase-change process known in the literature as Stefan problem (we remark that a more appropriate name would be Lamé-Clapeyron-Stefan problem). They are concerned with theoretical, numerical and experimental methods and also with various possible applications.

Together with the term "Stefan problem", the term phase-change problem, melting or freezing problem, fusion or solidification problem, moving or free boundary problem, Stefan-like problem are used, according to the particular field being studied.

The author's purpose in writing this bibliography is to provide usable information in the field of M-FBP for the H-DE both for the theoretical and the applied aspects.

The collection of titles began in 1977. As a result of the systematic organization of the material accumulated, a first bibliography (with 773 references) on M-FBP for the H-DE appeared in 1981. These 644 papers (of the quoted 773 titles) were classified into three main branches (theoretical, numerical and experimental) each containing several sub-sections, according to the following plan:

- I. Moving boundary-problems for the heat equation
  - I.1. One-dimensional case
  - I.2. Multidimensional case
  - I.3. Physical applications
  - I.4. Application to free boundary problems
- II. Free boundary problems for the heat equation
  - II.1. Free boundary problems of Stefan type
    - II.1.1. One-dimensional case
      - II.1.1.1. One-phase problem (theoretical, numerical methods and applications)
      - II.1.1.2. Two-phase problem (theoretical, numerical methods and applications)
    - II.1.2. Multidimensional case
      - II.1.2.1. One-phase problem (theoretical, numerical methods and applications)
      - II.1.2.2. Two-phase problem (theoretical, numerical methods and applications)
  - II.1.3. Other generalities

- II.1.3.1. Free boundary problems in a gaseous state
- II.1.3.2. Experimental works
- II.1.3.3. Solid-liquid interphase
- II.1.3.4. Other applications
- II.2. Free boundary problems not of Stefan type
  - II.2.1. Diffusion-consumption of oxygen in absorbing tissue
  - II.2.2. Flow of two immiscible fluids in a porous medium
  - II.2.3. Movement of a compressible fluid through a porous medium
  - II.2.4. Impact of a viscoplastic bar on a rigid obstacle
  - II.2.5. Chemical reactions between two substances
  - II.2.6. Other free boundary problems for the heat equation
    - II.2.6.1. Of an implicit type
    - II.2.6.2. Of an explicit type

Our aim is now to continue this work on the same line, including a new collection of titles.

## II. SOME GENERAL REMARKS.

### NOTE 1:

To avoid confusion between the terms "free boundary" and "moving boundary", we think it is advisable to point out the difference between them, especially since both terms are used indiscriminately in the literature (see e.g., the International Symposiums or Conferences on this subject).

On the other hand, in Cryer (1978) the author discusses the relationship between moving boundary problems (parabolic and time-dependent) and free boundary problems (elliptic and steady state). Because of this definition, approximately 1% of the references (namely 53) in his bibliography on free boundary problems [Cryer (1977)] is concerned with heat conduction and diffusion (see 1.6).

Our definition follows the one frequently used e.g. in the Italian literature (Fasano-Primicerio's group). In general, the problems given for the heat or diffusion equation are classified in the following way:

- boundary problems: - fixed**
- moving**
- free (implicit type or explicit type)**

The fixed boundary problems (FiBP) for the heat equation are those studied in the domain  $(x_1, x_2) \times (0, T)$ , i.e., the classical problems analyzed in any basic course of partial differential equations, such as:

(FiBP)	i) $u_t - u_{xx} = f(x, t),$		$x_1 < x < x_2, 0 < t < T,$
	ii) $u(x, 0) = h(x)$		$x_1 \leq x \leq x_2,$
	iii) $u(x_1, t) = f_1(t)$	or	$u_x(x_1, t) = f_1(t), 0 < t < T,$
	iv) $u(x_2, t) = f_2(t)$	or	$u_x(x_2, t) = f_2(t), 0 < t < T,$

which are not included in our bibliography and analysis.

The moving boundary problems (MBP) for the heat equation are those studied e.g., in the domain  $\{(x, t) / s_1(t) < x < s_2(t), 0 < t < T\}$  with  $s_1(t) < s_2(t)$ , functions given in  $(0, T)$ , i.e., the spatial domain of the unknown function varies with time because of a law of movement, known a priori, such as

$$\begin{array}{ll}
 \text{(MBP)} & \begin{array}{ll}
 \text{i) } u_t - u_{xx} = f(x, t), & s_1(t) < x < s_2(t), 0 < t < T, \\
 \text{ii) } u(x, 0) = h(x), & s_1(0) \leq x \leq s_2(0), \\
 \text{iii) } u(s_1(t), t) = f_1(t) & \text{or } u_x(s_1(t), t) = f_1(t), 0 < t < T, \\
 \text{iv) } u(s_2(t), t) = f_2(t) & \text{or } u_x(s_2(t), t) = f_2(t), 0 < t < T.
 \end{array}
 \end{array}$$

Moreover, the domain can be of the form  $\{(x, t) / x < s(t), 0 < t < T\}$  or  $\{(x, t) / s(t) < x, 0 < t < T\}$ . All these problems were originally studied at the beginning of the XX century by Gevrey, Goursat, Holmgren, Levi, etc., and of course there exists an enormous bibliography on this subject. We refer e.g. to J.R. Cannon's book (1984).

The free boundary problems (FBP) for the heat equation are those in which the spatial domain of the unknown function varies with time because of a law of movement not known a priori. The fact of not knowing the boundary or part of it, determines, of course, the mathematical need to impose new condition on the unknown function, which will depend on the physical problems studied. In general, the new condition to be imposed on the unknown function is deduced from the principle of conservation of energy across the boundary. Thus it follows that this boundary is the complementary unknown of the problem, and is called free boundary of the problem under analysis.

One of the most important FBP for the H-DE is the so-called Stefan problem (Lamé-Clapeyron (1981), Stefan (1989-1991)). Its mathematical formulation, in a dimensionless form, is given by: Find  $T > 0$ ,  $u = u(x, t)$  and  $x = s(t)$  such that they satisfy, e.g., the following conditions

$$\begin{array}{ll}
 \text{(FBP)} & \begin{array}{l}
 \text{i) } u_t - u_{xx} = 0, \quad 0 < x < s(t), 0 < t < T, \\
 \text{ii) } s(0) = b, \\
 \text{iii) } u(x, 0) = h(x), \quad 0 \leq x \leq b, \\
 \text{iv) } u(s(t), t) = 0, 0 < t < T, \\
 \text{v) } u_x(s(t), t) = -s'(t), 0 < t < T, \\
 \text{vi) } u(0, t) = f(t) \quad \text{or} \quad u_x(0, t) = f(t), 0 < t < T.
 \end{array}
 \end{array}$$

The condition (FBPv) is called the **Stefan condition**.

Many free boundary problems for the heat equation can be divided into two classes, the explicit and the implicit types, according to whether the speed of the free boundary appears explicitly, in the conditions imposed on this boundary. That is to say, if the free boundary is given by  $x = s(t)$ , then the problem will be of an explicit (implicit) type if derivative of  $s(t)$  appears (does not appear) in the condition imposed on  $x = s(t)$ . An example of a (FBP) of explicit type is the classical Stefan problem and an example of the implicit type is the diffusion-consumption of oxygen in a living tissue (Crank-Gupta (1972)). The free boundary conditions for the last problem are given by:

$$u(s(t), t) = 0, \quad 0 < t < T,$$

$$u_x(s(t), t) = 0, \quad 0 < t < T.$$

In general, free boundary problems of explicit and implicit types are related to each other [Schatz (1969), Fasano (1974)].

**NOTE 2:**

Among the FBP for the H-DE we may have Stefan problem, diffusion-consumption of oxygen in a living tissue, noncatalytic gas-solid diffusion-reaction problem, penetration of solvents into glassy polymers, continuous casting problem and other solidification processes, ground freezing, ablation by melting, welding two steel plates, the shape of laser melt pools, electromechanical machining, Hele-Shaw flow, solidification of binary alloys, storage of solar energy, porous media, fresh and salt groundwater, supercooling and superheating effects, freezing of foodstuffs, etc.

In order to have an idea of the importance of the methods and applications related to F-MBP for the H-DE, we may mention :

i) **Conferences, meetings or seminars completely devoted to the subject:** [Albretch-Collatz-Hoffmann (1982), Antonsev-Hoffmann-Khludnev (1992), Bossavit-Damlamian-Frémond (1985), Chadam-Rasmussen (1993), Delfour (1992), Diaz-Herrero-Liñan-Vazquez (1995), Fasano-Primicerio (1983), Friedman-Spruck (1993), Hoffmann (1977), Hoffmann-Sprekels (1990), Magenes (1980), Neittaanmaki (1991), Niezgodka-Pawlow (1985), Niezgodka-Strzelecki (1996), Ockendon-Hodgkins (1975), Rodrigues (1989), Wilson-Solomon-Boggs (1978), Wrobel-Brebbia (1991, 1993), Wrobel-Brebbia-Nowak (1990, 1992, 1994), Wrobel-Sarler-Brebbia (1995)]. Moreover, didactic seminars were given in [Fasano-Primicerio (1986), Tarzia (1984, 1987, 1989, 1993, 1995)].

ii) **Books or booklets exclusively devoted to this subject:** [Alexiades-Solomon (1993), Anantharaman-Suryanarayana (1987), Carey (1992), Crank (1984), Datzeff (1970), Elliott-Ockendon (1982), Fasano (1987, 1989), Gurtin (1993), Hill (1987), Kurz-Fisher (1986, 1992), Lock (1996), Lunardini (1991), Meirmanov (1992), Primicerio-Gianni (1989), Rubinstein (1971), Shyy-Udaykumar-Rao-Smith (1996), Tarzia (1987), Ughi (1991), Visintin (1996), Vuik (1993), Yamaguchi-Nogi (1977), Zerroukat-Chatwin (1994)].

iii) **Books that devote several chapters or sections to the subject:** [Arpaci (1966), Aziz-Na (1984), Baiocchi-Capelo (1978), Barbu (1984), Barenblatt (1979, 1993, 1996), Bensoussan-Lions (1978), Biot (1970), Boley-Weiner (1960), Buckmaster-Ludford (1983), Caffarelli-Cabre (1995), Cannon (1984), Carslaw-Jaeger (1959), Chalmers (1984), Chipot (1984), Colton (1976, 1988), Crank (1956), Diaz (1985), Duvaut-Lions (1972), Eckert-Drake (1959, 1972), Fife (1988), Flemings (1974), Friedman (1964, 1982, 1988), Froment-Bischoff (1979), Goursat (1927), Grisvard (1985), Hill (1982), Jerome (1983), Kenig (1994), Kinderlehrer-Stampacchia (1980), Ladyzenskaja-Solonnikov-Ural'ceva (1968), Levi (1960), Lions (1969, 1976), Luikov (1968), Lunardini (1981), Mellor (1978), Meyer (1973), Mori (1986), Naumann (1984), Neittaanmaki-Tiba (1994), Ockendon-Taylor (1983), Ozisik (1980), Rodrigues (1987), Rosenberger (1979), Rubinstein-Rubinstein (1998), Shukla (1990), Smirnov (1964), Sokolowski-Zolesio (1992), Szekely-Evans-Sohn (1976), Szekely-Themelis

(1971), Tarzia (1981), Tayler (1986), Tiba (1990), Tikhonov-Samarskii (1963), Weber (1901), Wilmott-Dewynne-Howison (1993, 1995), Woodruff (1973), etc.].

iv) **Review papers on the subject both from the theoretical and/or numerical point of view:** Aronson (1986), Bankoff (1964), Barbu (1985), Biloni (1978), Broadbridge (1992), Chapman-Howison-Ockendon (1992), Crank (1981), Danilyuk (1985), Duvaut (1976), Eckert (several times on heat transfer literature), Fasano (1987, 1998), Fox (1975), Friedman (1979), Furzeland (1980), Garguichevich-Sanzuel (1984), Guilding (1982), Goodman (1964), Hoffmann-Niezgodka (1983), Luikov (1971), Magenes (1976, 1981, 1998), Meyer (1978, 1983), Muehlbauer-Sunderland (1965), Niezgodka (1984), Nochetto (1984), Peletier (1973), Philip (1969), Primicerio (1973, 1981), Quilghini (1975), Rodrigues (1987, 1989), Rubinstein (1980), Saguez (1981), Sestini (1960), Stakgold (1986), Tarzia (1984, 1986), Verdi (1987, 1998), Villa (1984), Visintin (1998), etc.].

v) **A long bibliography on the subject** can be found in Broadbridge-White (1988), Danilyuk (1985), Goodman (1964), Kenmochi (1981), Nitsche (1980), Philip (1969), Primicerio (1981), Senf (1984), Tarzia (1981, 1984, 1988), Verdi (1998), Visintin (1998), Wilson-Solomon-Trent (1979), and in other books and review papers cited above].

### NOTE 3:

i) Active research during the past three decades has produced a variety of methods for finding theoretical, numerical and experimental results as well as important practical applications.

The number of titles on M-FBP for the H-DE has been exponentially increasing during the past years; an analysis of this fact is given in Table I:

<b>Period</b>	<b>Numbers of titles of the present bibliography which have been published</b>
• 1831	1 (G. Lamé - B.P. Clapeyron)
• 1889-1891	6 (J. Stefan)
• 1901-1930	14
• 1931-1950	19
• 1951-1960	103
• 1961-1970	341
• 1971-1980	1119
• 1981-1990	2107
• 1991-1998/9	2159
<b>Total</b>	<b>5869</b>

ii) The 5869 titles of our bibliography have been published in the following form (Table II):

<b>Type of publication</b>	<b>Number of references</b>
• Scientific journals	4071
• Papers published in a Symposium (edited as a book or a collection)	1282
• Technical reports	247
• Books which have at least a chapter or a section on M-FBP for the H-DE	122
• Edition of a Symposium which has at least 3 articles on M-FBP for the H-DE	88
• Thesis	59
<b>Total</b>	<b>5869</b>

iii) Table of the publications having at least one of our 5869 titles (Table III):

<b>Kid of publication</b>	<b>Quantity</b>
• Scientific journals	588
• Books which include a chapter or some sections related to M-FBP for the H-DE	98
• Technical reports	56
• Proceedings of symposia not specifically on M-FBP but which include at least 3 papers on M-FBP for the H-DE	53
• Proceedings of conferences on M-FBP for the H-DE	35
• Collection	30
• Books on M-FBP for the H-DE	24
<b>Total</b>	<b>884</b>

iv) English, French, Italian, Portuguese and Spanish are the scientific languages better known by the author. This may have affected the selection of references. Original references in Russian are not, in general, included in this work (See point (vi) in Note 4).

The 5869 titles of our bibliography have been written in the following languages (Table IV):

<b>Language</b>	<b>Number of titles written in</b>	<b>There exists translation in English</b>	<b>Number of titles available in English</b>
• English	5119	---	5119
• Russian	209	208	208
• Spanish	192	5	15
• French	165	4	4
• German	70	1	1
• Italian	61	1	1
• Japanese	25	24	24
• Chinese	---	11	11
• Portuguese	10	1	1
• Rumanian	4	---	---
• Polish	2	1	1
• Finish	1	---	---
<b>Total</b>	<b>5869</b>	<b>266</b>	<b>5385</b>

We remark that 92% of our bibliography is available in the English language.

v) The 32 physical-engineering scientific journals which have published a substantial number of titles (with more than 13 papers) are (Table V):

Scientific journal (Abbreviation)	Number of titles published in the present bibliography
• Int. J. Heat Mass Transfer	372
• J. Heat Transfer	206
• Int. Comm. Heat Mass Transfer (Letters Heat Mass Transfer)	91
• J. Crystal Growth	78
• Int. J. Numer. Meth. Eng.	69
• Numer. Heat Transfer	61
• Chem. Eng. Sci	51
• Heat Transfer-Soviet Research	35
• Int. J. Eng. Sci.	29
• J. Comput. Physics	29
• Soviet Phys. Dokl.	29
• Water Resour. Res.	29
• J. Appl. Phys.	27
• Metallurg. Tech.	27
• Metallurg. Mater. Trans.	25
• Latin Amer. Appl. Res. (Latin Amer. J. Heat Mass Transfer, Latin Amer. J. Chem. Eng. Appl. Chem.)	22
• AICh. E Journal	20
• Comp. Meth. Appl. Mech. Eng.	20
• AIAA Journal	18
• Appl. Sci. Res.	18
• A. I. Ch. E. Symp. Series	17
• Adv. Water Resour.	17
• J. Fluid Mech.	17
• Physica (*)	17
• Trans.Metallurg.Soc.AIME	16
• Metallurg. Trans.	16
• J. Appl.. Mech.	14
• Mech. Res. Comm.	14
• Soviet Phys. Tech. Phys.	14
• Nuclear Sci. Eng.	13
• Soil Sci.	13
• Soil Sci. Soc. Amer. J.	13
<b>Total</b>	<b>1420</b>

They have published 35% of the 4039 titles that appeared on scientific journals. We remark that 10 titles (of the 17 (\*)) published on Physica are due to the edition of a Symposium (Vol. 12D (1984)).

vi) The 36 mathematical scientific Journals which have published a substantial number of titles (with more than 13 papers) are (Table VI):

Scientific Journal (Abbreviation)	Number of titles published in the present bibliography
• SIAM J. Math. Anal.	90
• Quart. Appl. Math.	87
• IMA J. Appl. Math.(J. Inst. Math. Appl.)	76 (**)
• Nonlinear Anal. Th. Meth. Appl.	72
• J. Math. Anal. Appl.	66
• Euro. J. Appl. Math.	65
• SIAM J. Appl. Math.(J. SIAM)	56
• J. Diff. Eq.	55
• Arch. Rat. Mech. Anal.	52
• Boll. Un. Mat. Italiana	47
• USSR Comput. Maths. Math. Phys.	46
• Soviet Math. Dokl.	39
• Indiana Univ. Math.J. (J. Math. Mech.)	39
• C. R. Acad. Sc. Paris	38
• SIAM J. Numer. Anal.	37
• Ann. Mat. Pura Appl.	36
• Control Cybernet.	35 (***)
• Trans. Amer. Math. Soc.	35
• Comm. Partial Diff. Eq.	34
• Math. Mech. Appl. Sci.	30
• Z. Angew. Math. Mech.	29
• Quart. J. Mech. Appl. Math.	28
• J. Inst. Math. Appl.	25
• Diff. Eq.	24
• Ann. Sc. Norm. Sup. Pisa	18
• Meccanica	18
• IMA J. Numer. Anal.	17
• J. Eng. Math.	17
• Numer. Math.	16
• Proc. Royal Soc. Edinburgh	16
• Comm. Pure Appl. Math.	15
• J. Appl. Math. Mech.	15
• Adv. Math. Sci. Appl.	14
• Appl. Anal.	14
• Rocky Mountain J. Math.	13
• Proc. Royal Soc. London	13
<b>Total</b>	<b>1327</b>

They have published 33% of the 4039 titles that appeared on scientific journals. We remark that 11 titles (of the 76 (\*\*)) published on IMA J. Applied Mathematics are due to the edition of a Conference (Vol. 35 (1985)) and that 15 titles (of the 35 (\*\*\*)) published on Control and Cybernetics are due to the edition of a Conference (Vol. 14, No. 1-3 (1985)).

vii) The collections which have published a substantial number of titles are (Table VII):

Name of the collection	Number of titles published in the present bibliography
<ul style="list-style-type: none"> <li>• Pitman Research Notes in Math. Series</li> <li>• Int. Series Numer. Math. (ISNM)</li> <li>• Ground Freezing</li> <li>• Permafrost</li> <li>• Cuadern. Inst. Mat. B. Levi</li> <li>• Moving Boundary Problems (Comp. Mech.)</li> <li>• Computational Heat Transfer</li> <li>• Mecánica Computacional</li> <li>• Numerical Heat Transfer</li> <li>• Free Univ. Berlin</li> <li>• Gakuto Int. Series: Math. Sciences Appl.</li> <li>• Heat Transfer</li> <li>• ECMI - European Conf. Industrial Math.</li> <li>• Solidification</li> <li>• Congreso Latinoam. Transf. Calor y Materia</li> <li>• Lecture Notes in Math.</li> <li>• Lecture Notes in Control &amp; Inform. Sciences</li> <li>• Drying</li> </ul>	<p style="text-align: right;">286 93 86 85 65 56 39 39 27 22 21 21 19 18 17 16 14 8</p>
<b>Total</b>	<b>937</b>

They have published 94% of the 991 titles that appeared on 30 collections. We remark that there exists at least 27 others collections which have published a few numbers of papers on the subject and this fact has not been considered for the last statistics.

We remark that 266 (over 286) titles published in the collection Pitman Res. Notes in Math. Series (Addison Wesley - Longman) (see Table IX) refers only to the papers contained in the following eleven volumes #: 78, 79, 120, 121, 185, 186, 280, 281, 282, 323, 363 corresponding to the Proceedings of the International Conferences on Free Boundary Problems.

viii) The technical reports including a substantial number of titles are (Table VIII) (We remark that the technical reports published in a scientific journal, proceeding or collection, etc. were not considered here):

<b>Technical reports</b>	<b>Number of titles published in the present bibliography</b>
• Publ. Ist. Analisi Numerica, Pavia	42
• Math. Res. Center, Univ. Winsconsin	19
• ORNL, Oak Ridge	15
• INRIA, Rocquencourt	14
• Sem. Mat., Univ. Brescia	13
• CNEA, Buenos Aires	12
• Dip. Mat."F. Enriques", Univ. Milano	11
• ICTP, Trieste	10
• Dip. Mat."U.Dini", Univ.Firenze	9
• IMA, Univ. Minnesota	8
• Conf. Sem. Mat., Univ. Bari	6
• CMAF, Univ. Lisboa	6
• Chiba Univ.	5
• IAM-UBA, Univ. Buenos Aires	5
• Univ. Bonn	5
• Dept. Math., Univ.Paris-Sud, Orsay	5
<b>Total</b>	<b>185</b>

They have published 75 % of 247 titles that appeared as a technical report.

ix) There are 210 titles in our bibliography (122 books and 88 proceedings which have at least three papers on the subject) which have been published by a Publisher. Among them, we may cite (Table IX):

<b>Publisher</b>	<b>Number of titles (books and proceeding)</b>
• Addison Wesley-Longman (ex Pitman)	17
• Springer Verlag	15
• John Wiley	13
• Academic Press	11
• Birkhäuser Verlag	9
• Computational Mechanics Publ.	9
• AMCA, Santa Fe	8
• Clarendon Press	8
• Pergamon Press	8
• Universidad Nacional de Rosario	8
• Dunod-Gauthier Villars	7
• McGraw Hill	7
• Cambridge University Press	6
• American Mathematical Society	4
<b>Total</b>	<b>130</b>

They include the 62 % of the 210 titles which were published by a Publisher.

**NOTE 4:**

Some abbreviations and conventions has been used, e.g. :

- i) The bibliography is arranged alphabetically by the first author. A prefix is treated as part of the name so that e.g. Van Duyn is listed under V and not under D. Names beginning with, e.g., Mac, Mc and Di are listed under MAC, MC and DI respectively.
- ii) In general, the source of information (authors, titles, references, etc.) is indicated.
- iii) When a non-English paper has an English translation, we quote, in general, the English version.
- iv) Abbreviations of names of journals are those used in the current literature whenever they are available.
- v) We have only considered as the year of the publication the one appearing in the source reference and not the year of the conference or journal reception. Moreover, when the publication year is indicated e.g. (1980/81) it was considered like (1980). This is criterion followed in preparing Table I.
- vi) **Important:** Russian (in general, non-western languages) literature on this field (which has not been translated to the English language) will be introduced in forthcoming issues.

**NOTE 5:**

For the search of bibliography many scientific libraries were consulted during 1977-1999; Among them, I must mention the following:

- i) Hemeroteca del Instituto de Matemática "Beppo Levi", Univ. Nac. de Rosario, Rosario (Argentina),
- ii) Biblioteca dell'Dipartimento di Matematica "Ulisse Dini", Univ. di Firenze, Firenze (Italy),
- iii) Bibliothèque de Mathématiques Recherche, Univ. de Paris VI, Paris (France),
- iv) Centre de documentation de l'INRIA, Rocquencourt (France),
- v) Bibliothèque de Physique Recherche, Univ. de Paris VI, Paris (France),
- vi) Hemeroteca del CERIDE, Santa Fe (Argentina),
- vii) Biblioteca do IMPA e do LNCC, Rio de Janeiro (Brazil),
- viii) Hemeroteca del Departamento de Matemática (UBA) y del IAM, Buenos Aires (Argentina),
- ix) Bibliothèques de Mécanique Théorique, d'Analyse Numérique, de Chimie Recherche et de Sciences de la Terre, Univ. de Paris VI, Paris (France),
- x) Library of ICTP, Trieste (Italy),
- xi) Biblioteca del Grupo de Frontera Libre GIFLA (CONICET-UA), Univ. Austral, Rosario (Argentina).

**NOTE 6:**

Undoubtedly, I am responsible for the mistakes and omissions that might have slipped in the text. For those I apologize. I would welcome information about them so that they could be corrected in a possible future edition.

I would greatly appreciate and would like to receive reprints, technical reports, proceeding, books, thesis, preprints or even references or photocopies of articles on M-FBP which have not been included in this bibliography and also some hints for new research. These could be incorporated in a future edition.

It is my hope that this bibliography will reduce the duplication of effort and stimulate the cross-fertilization of ideas among the different and numerous disciplines concerned with M-FBP. Finally, I also hope that this long bibliography will be useful to researchers in

moving and free boundary problems in partial differential equations particularly regarding the heat-diffusion equation.

**NOTE 7 (Service to the International Scientific Community)**

I have a copy of all the references mentioned in the present bibliography. Any person who for any reason has not been able to find any of the references could ask for a copy (in general, an interchange basis will be offer).

**Acknowledgements:**

This work has been partially sponsored by CONICET and Universidad Austral (Argentina). This financial support was granted to the Research Project "Free Boundary Problems for the Heat-Diffusion Equation" (Problemas de Frontera Libre para la Ecuación del Calor-Difusión).

I am grateful to the Institutions that have granted me fellowships, which were developed in: G.N.F.M. of the Italian CNR, ICTP and Univ. di Firenze (Italy); Univ. de Paris VI and INRIA (France); and LNCC (Brazil).

Thanks to my colleagues and friends Mary Dzioba, Graciela Garguichevich, Riccardo Ricci, Cristina Turner and Luis Villa for your help and interest in this work during the last two decades. Special thanks to Professors A. Fasano and M. Primicerio for their constant help during the last seventeen years and for having published my previous bibliography in 1988.

Thanks to Cristian Conde, Evangelina Fernández, Ariel Lombardi and Mariela Olguín for their help in the writing and organization of the data base files during different periods in the last decade. Special thanks to Lorena Duarte, Adolfo Giancarelli, Ariel Gulisano y Diego Gregoraz for their help in the printing of the bibliographical data-base and the web publication.

Finally, I am grateful to the great number of people (researchers, librarians, etc.) and Institutions that have silently contributed to this work.

I dedicate this work to Norma and my sons María Silvina and Pablo Alberto.

Domingo Alberto Tarzia  
Rosario, octubre de 1999.

### **III. REFERENCES**

There are 5869 references classified in the following way:

1. Books or booklets exclusively devoted to the subject (with 24 titles).
2. Thesis on the subject (with 59 titles).
3. Books that devote several chapters or sections to the subject (with 98 titles)
4. Editors of conferences, meetings or seminars completely devoted to the subject (with 35 titles).
5. Papers published on conferences, meetings or seminars completely devoted to the subject (with 622 titles).
6. Editors of a symposium not specifically on the subject but which includes at least three papers on the subject (with 53 titles).
7. Papers published on a symposium not specifically on the subject but which includes at least three papers on the subject (with 348 titles).
8. Papers on the subject published in a symposium not considered before (with 312 titles).
9. Papers on the subject published on scientific journal or some collections (with 4039 titles).
10. Papers on the subject to appear on scientific journal (with 32 titles).
11. Technical reports on the subject (with 247 titles).

All titles included in the eleven different data bases can be found in:

[www.austral.edu.ar/MAT-SerieA/2\(2000\)](http://www.austral.edu.ar/MAT-SerieA/2(2000))